## Factoring Trinomials: $x^2 + bx + c$

In another chapter we learned to use the FOIL method to multiply two binomials. In many cases the simplified form of the product was a trinomial. In this section we will learn to factor trinomials by reversing the FOIL method. In particular, we will focus on factoring trinomials on one variable w leading coefficient 1.

### Factoring Trinomials w Leading Coefficient 1

Using the FOIL method to multiply $(x+5)$ & $(x+3)$, we find the following.

$$(x+5)(x+3) = \overset{F}{x^2} + \overset{O}{3x} + \overset{I}{5x} + \overset{L}{3\cdot 5} = x^2 + \underset{3+5}{8x} + \underset{3\cdot 5}{15}$$

We see that the leading coefficient (coefficient of $x^2$) is 1, the coefficient 8 is the sum of 5 & 3, & the constant term 15 is the product of 5 & 3.

In general, this can be expressed as follows.

$$\begin{aligned}(x+a)(x+b) &= x^2 + bx + ax + ab \\ &= x^2 + (b+a)x + ab\end{aligned}$$

Now, given that a trinomial w leading coefficient 1, we want to find the binomial factors, if any. Reversing the relationship between a & b, as shown above, we can proceed as follows.

To factor a trinomial w leading coefficient 1, find two factors of the constant term whose sum is the coefficient of the middle term. (If those factors do not exist, the trinomial is not factorable).

For example, to factor $x^2 + 11x + 30$, we need positive factors of $+30$ whose sum is $+11$.

| Positive Factors of 30 | | Sums of These Factors |
|---|---|---|
| $1\cdot 30$ | → | $1+30=31$ |
| $2\cdot 15$ | → | $2+15=17$ |
| $3\cdot 10$ | → | $3+10=13$ |
| $5\cdot 6$ | → | $5+6=11$ |

Because $5 \cdot 6 = 30$ & $5 + 6 = 11$, we have

$$x^2 + 11x + 30 = (x + 5)(x + 6)$$

Listing all pairs is not necessary. This stage is called the trial-and-error stage. That is, you can try different pairs until you find the correct pair. If a pair w the required product & sum does not exist, the polynomial is not factorable.